

# *BASIC MATH*

*A Self-Tutorial*

*by*

*Luis Anthony Ast*

*Professional Mathematics Tutor*

*LESSON 2:*

*EQUALITIES & INEQUALITIES*

**VideoMathTutor.com**

*Copyright © 2005*


All rights reserved. No part of this publication may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopy, recording, or any information storage or retrieval system, without permission in writing of the author.

E-mail may be sent to: [Luis-Ast@VideoMathTutor.com](mailto:Luis-Ast@VideoMathTutor.com)

# ORDER OF NUMBERS

Numbers are ordered relative to where they are in relation to zero. Numbers to the *right* of zero (on the number line) are called Positive numbers and those to the *left* are Negative numbers.

|  
Negative 0 Positive

 **ON THE CALCULATOR...** To type in negative numbers, use the negation key  $\ominus$  and the number keys:  $\boxed{1}$ ,  $\boxed{2}$ ,  $\boxed{3}$ ,  $\boxed{4}$ , etc. Use the  $\boxed{0}$  key for zero, but you should NOT type the  $\boxed{+}$  key in front of a number when you want to input a positive number into the calculator. Just type the number. For example, To type in the expression:  $+3 - 4$  you should press the following keys:  $\boxed{3} \boxed{-} \boxed{4} \boxed{\text{ENTER}}$ . *<The answer is: -1>*. Don't type:  $\boxed{+} \boxed{3} \boxed{-} \boxed{4} \boxed{\text{ENTER}}$ . You will get all sort of strange results with this.

Of course, this does not apply to addition of numbers. To type  $1 + 2$  you type:  $\boxed{1} \boxed{+} \boxed{2} \boxed{\text{ENTER}}$ .

## *EQUA = LITIES*


A number is Equal to another if it is located at the *same* point as another number on the real number line. For example, the number “*a*” is equal to the number “*b*” if they can be graphed on the same point on a real number line:

*a*

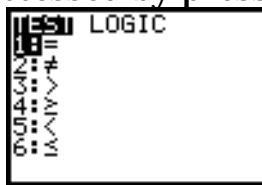


*b*

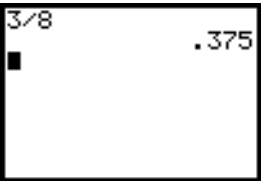
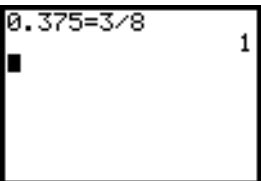
The above represents:  $a = b$

 **ON THE CALCULATOR...** On simple, non-graphing calculators, the  $\boxed{=}$  key is used to have the calculator figure out what is equal to the expression you typed into it. For graphing calculators, use the  $\boxed{\text{ENTER}}$  key. There *is* an equals symbol “=” that is available on graphing calculators, but it is used

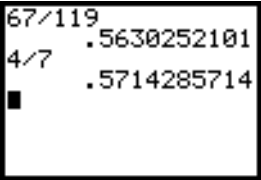
for comparisons between two sides of an expression. This key can be found in the “TEST” Menu that is accessed by pressing **[2nd] ↑TEST ↓**



**PROBLEM 1:** Is 0.375 equal to  $\frac{3}{8}$ ?

What to Do:	On the Calculator Screen:
<p><b>The Easy Way:</b> Just type in the <math>\frac{3}{8}</math> and see if it equals to 0.375:</p> <p><b>[CLEAR]</b> <i>&lt;It is a good habit to CLEAR the display before starting new computations on the calculator&gt;</i></p> <p><b>[3] [÷] [8]</b> <b>[ENTER]</b></p>	 <p>Yep! It's the same!</p>
<p><b>The More Complicated Way:</b> Compare 0.375 to <math>\frac{3}{8}</math> using the “=” in the TEST Menu:</p> <p><b>[CLEAR]</b></p> <p><b>[0] [.] [3] [7] [5] [2nd] ↑TEST ↓ [ENTER]</b> <b>[3] [÷] [8]</b> <b>[ENTER]</b></p> <p> <i>By The Way...</i> The more complicated way is used mostly when creating programs, so don't worry about it too much.</p>	 <p>The result of “1” means you typed in a TRUE statement, so the numbers are equal. If the result had been “0,” then it would have been FALSE, and the numbers would have been not equal.</p> <p><i>&lt;The slash through the zero is used so as not to confuse the display with the letter O, which looks like the number 0&gt;</i></p>

**PROBLEM 2:** Is  $\frac{67}{119}$  equal to  $\frac{4}{7}$ ?

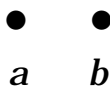
What to Do:	On the Calculator Screen:
<p><b>[CLEAR]</b></p> <p><b>[6] [7] [÷] [1] [1] [9]</b> <b>[ENTER]</b></p> <p><b>[4] [÷] [7]</b> <b>[ENTER]</b></p>	 <p>No, they are not equal.</p>

# INEQUA < ITIES

◆ A number is **Less Than** another if *its* point on the number line is to the *left* of the other.

Math Symbol for “is less than” is: <

For example, number “*a*” is less than a number “*b*” ( $a < b$ ) if the point representing *a* can be graphed to the left of point where *b* is on the real number line:

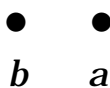


The above represents:  $a < b$  *<a is less than b>*

◆ A number is **Greater Than** another if it is to the *right* of the other number.

Math Symbol for “is greater than” is: >


For example, number “*a*” is greater than a number “*b*” ( $a > b$ ) if the point representing *a* can be graphed to the right of point where *b* is on the real number line:

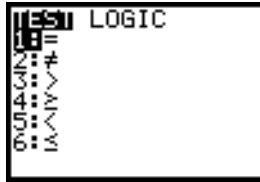


The above represents:  $a > b$  *<a is greater than b>*



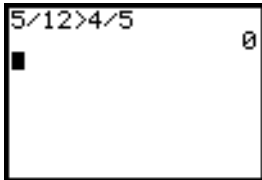
It’s easy to “mix up” the symbols, so just remember that the “opening” of the symbol is always towards the larger value. The “point” is towards the smaller value. Another way to remember is that you can use the “<” to “write” the words “<ess than.” The “less than” symbol looks like the letter “L.” You can’t do this with the other symbol.


 **ON THE CALCULATOR...** You can access the inequality keys by accessing the “TEST” Menu. Press: **[2nd] ↑ TEST ↓** to see the following screen again:



Try using this screen for the following problem:

 **PROBLEM 3:** Is  $\frac{5}{12} > \frac{4}{5}$ ?

<i>What to Do:</i>	<i>On the Calculator Screen:</i>
CLEAR 5 ÷ 1 2 2nd ↑ TEST ↓ 3 4 ÷ 5 ENTER	 <p>Since the result is zero, this means we get a false statement; therefore, <math>\frac{5}{12}</math> is NOT greater than <math>\frac{4}{5}</math>.</p>

 *By The Way...* To solve the previous problem “by hand,” you would need to convert the fractions to decimals, then compare them.



Many symbols are used to compare one quantity with another. Here is a table of the most common ones, along with how to read the symbols, and how they are used in possible word problems:

	<i>How to say:</i>	<i>How used in word problems:</i>	<i>Example:</i>
=	“is equal to”	“is,” “is equal to” or “was”	$2 + 2 = 4$
$\neq$	“is not equal to”	Not usually in word problems. Used mostly to check validity of problem or to make a restriction.	Given: $\frac{1}{x}$ , then $x \neq 0$
$\approx$	“is approximately equal to”	Used to change exact answer to an approximate answer.	$\pi \approx 3.14$ $\sqrt{2} \approx 1.41$
<	“is less than”	“is less than” or “fewer than”	$3 < 5$
>	“is greater than”	“is greater than” or “more than”	$-2 > -7$
$\leq$	“is less than or equal to”	“no more than” or “at most”	“You can work at most 40 hrs/wk.” $W \leq 40$
$\geq$	“is greater than or equal to”	“no less than” or “at least”	“You must be at least 21 to enter here.” $A \geq 21$

Now let us put all this together in the next section.

# [INTERVAL], INEQUALITY & {SET NOTATION}

Often, the results of math problems are a span of numbers on the number line. We call these areas Intervals. There are three principal ways of representing these values that appear on the graph of a real number line: Interval Notation, Inequality Notation and Set Notation.

*<Set Notation is also called  
"Set-Builder" Notation>*

## THE BUILDING BLOCKS:

If the “endpoint” *includes* the value, then a “solid circle” or “dot” is used: ●

If the “endpoint” *does NOT include* the value, then an “open circle” or “open dot” is used: ○

The number line is “shaded” (thicker line: **—**) where values are included.

A regular line ( ) is used to show the part of the number line that is not part of what we want.

When what we want extends to either the left indefinitely, or to the right indefinitely, we use the infinity symbol ( $\infty$ ) to represent this. We use  $-\infty$  for the left, and just  $\infty$  (which is positive) for the right side of the number line.



## MEMORIZE TABLE:

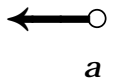

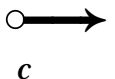
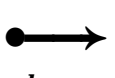

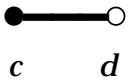



Here’s a handy little table that helps you remember how all the symbols are related:

<i>If you see</i> < <b>or</b> >	<i>If you see</i> ≤ <b>or</b> ≥
The “dot” on number line is “open” ○	The “dot” on number line is “closed” ●
Always use Parentheses: ( or )	Always use Square Brackets: [ or ]
When using the infinity symbols, <b>ALWAYS</b> use parentheses: ( $-\infty$ or $\infty$ )	




**MEMORIZE  
THIS TOO!**

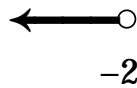
Here is a table of most of the basic intervals you will encounter: *<The x is just used as an example. Any variable may be used>*

Number Line Graph	Interval Notation	Inequality Notation	Set Notation	How to Read the Set Notation
	$(-\infty, a)$ <i>&lt;open interval&gt;</i>	$x < a$	$\{x \mid x < a\}$	The set of all $x$ 's such that $x$ is less than $a$
	$(-\infty, b]$ <i>&lt;half-open interval&gt;</i>	$x \leq b$	$\{x \mid x \leq b\}$	The set of all $x$ 's such that $x$ is less than or equal to $b$
	$(c, \infty)$ <i>&lt;open interval&gt;</i>	$x > c$	$\{x \mid x > c\}$	The set of all $x$ 's such that $x$ is greater than $c$
	$[d, \infty)$ <i>&lt;half-open interval&gt;</i>	$x \geq d$	$\{x \mid x \geq d\}$	The set of all $x$ 's such that $x$ is greater than or equal to $d$
	$(a, b)$ <i>&lt;open interval&gt;</i>	$a < x < b$	$\{x \mid a < x < b\}$	The set of all $x$ 's such that $x$ is in between $a$ and $b$
	$[c, d)$ <i>&lt;half-open interval&gt;</i>	$c \leq x < d$	$\{x \mid c \leq x < d\}$	The set of all $x$ 's such that $x$ is in between $c$ and $d$ , and includes $c$
	$(e, f]$ <i>&lt;half-open interval&gt;</i>	$e < x \leq f$	$\{x \mid e < x \leq f\}$	The set of all $x$ 's such that $x$ is in between $e$ and $f$ , and includes $f$
	$[g, h]$ <i>&lt;closed interval&gt;</i>	$g \leq x \leq h$	$\{x \mid g \leq x \leq h\}$	The set of all $x$ 's such that $x$ is in between $g$ and $h$ , inclusive <i>&lt;"inclusive" is used when BOTH endpoints are included&gt;</i>
 <i>&lt;all real numbers&gt;</i>	$(-\infty, \infty)$ <i>&lt;open interval&gt;</i>	Not Applicable	$\{x \mid x \in \mathbb{R}\}$	The set of all $x$ 's such that $x$ is an element of the set of real numbers



There are other number line graphs that are possible, but these are covered in the Algebra Lesson: "Solving Linear Inequalities."

 **A DETAILED EXAMPLE:** Say we are given the following number line graph:




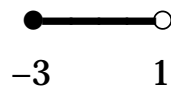
I will use  $x$  to represent the shaded part on the number line graph above. Now, let's figure out the interval, inequality and set notation associated with the given graph.

When creating the **interval notation**, always view it from the perspective "left-to-right." From the graph above, the interval notation describing it is:  $(-\infty, -2)$ . This is classified as an open interval, since parentheses are used at both ends of the interval.

For the **inequality notation**, note that all  $x$  values are less than (but not equal to) 2, since an "open dot" is used. The notation is:  $x < 2$ .

The **set notation** would look like:  $\{x \mid x < 2\}$  This is read as: "The set of all  $x$  values such that  $x$  is less than two."

 **ANOTHER EXAMPLE:** We are now given the following number line graph:



I will use  $x$  again to represent the shaded part on the number line graph above. Now, let's figure out the interval, inequality and set notation associated with the given graph.

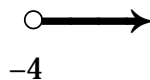
The **interval notation** describing it is:  $[-3, 1)$ . This is classified as a half-open interval, since a parenthesis is used only at one end of the interval.

For the **inequality notation**, note that all  $x$  values are between  $-3$  and  $1$ , but don't include  $1$ . To write the inequality notation, place the  $x$  between the two numbers. Use inequality symbols between the  $x$  and the numbers. The notation looks like:  $-3 \leq x < 1$ .

The **set notation** is:  $\{x \mid -3 \leq x < 1\}$  This is read as: "The set of all  $x$  values such that  $x$  is between negative three and one, and includes negative three."

# LESSON 2 QUIZ

① Using  $x$  to represent the shaded part on the number line graph below, write the interval, inequality and set notation associated with the given graph:



Interval Notation: \_\_\_\_\_

Inequality Notation: \_\_\_\_\_

Set Notation: \_\_\_\_\_

② Using  $x$  to represent the shaded part on the number line graph below, write the interval, inequality and set notation associated with the given graph:



Interval Notation: \_\_\_\_\_

Inequality Notation: \_\_\_\_\_

Set Notation: \_\_\_\_\_

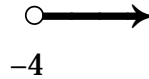
③ Fill in the following table:

<i>Number Line Graph</i>	<i>Interval Notation</i>	<i>Inequality Notation</i>	<i>Set Notation</i>
 $\longleftarrow \circ$ $6$			
	$[1, \infty)$		
		$-\frac{4}{11} < x \leq \frac{2}{3}$	
			$\{x \mid 0.6 \leq x < 5.9\}$

*ANSWERS ON NEXT PAGE...*

# ANSWERS

① Using  $x$  to represent the shaded part on the number line graph below, write the interval, inequality and set notation associated with the given graph:



Interval Notation:  $(-4, \infty)$

Inequality Notation:  $x > -4$

Set Notation:  $\{x \mid x > -4\}$

② Using  $x$  to represent the shaded part on the number line graph below, write the interval, inequality and set notation associated with the given graph:



Interval Notation:  $[0.3, 4.7]$

Inequality Notation:  $0.3 \leq x \leq 4.7$

Set Notation:  $\{x \mid 0.3 \leq x \leq 4.7\}$

③ Here is the filled table:

Number Line Graph	Interval Notation	Inequality Notation	Set Notation
 6	$(-\infty, 6)$	$x < 6$	$\{x \mid x < 6\}$
 1	$[1, \infty)$	$x \geq 1$	$\{x \mid x \geq 1\}$
 $-\frac{4}{11}$ $\frac{2}{3}$	$(-\frac{4}{11}, \frac{2}{3}]$	$-\frac{4}{11} < x \leq \frac{2}{3}$	$\{x \mid -\frac{4}{11} < x \leq \frac{2}{3}\}$
 0.6      5.9	$[0.6, 5.9)$	$0.6 \leq x < 5.9$	$\{x \mid 0.6 \leq x < 5.9\}$

## END OF LESSON 2